UNIVERSITY OF OSLO
Faculty of Mathematics and Natural Sciences

Exam in AST2110 — The Universe

Day of exam: Wednesday 8th June 2005

Exam hours: 14.30 – 17.30

This examination paper consists of 4 pages.

Appendices: None

Permitted materials: Rottmann: "Matematisk formelsamling"
Øgrim og Lian: "Størrelser og enheter i fysikk og teknikk"
Approved calculator
Two A4 pages (can be written on both sides) with your own notes

Make sure that your copy of this examination paper is complete before answering.

Exercise 1

A spaceship leaves the Earth at time $t = 0$ on its rectilinear journey towards the nearest star, Proxima Centauri, that is at a distance of 1.3 pc. The spaceship starts with velocity $v = 0$, and during the first part of the journey, it has constant acceleration $g = 9.8\,\mathrm{m\,s^{-2}}$, measured in the instantaneous inertial reference system of the spaceship. This part of the journey is part *i*.

When the spaceship is halfway to Proxima Centauri, the acceleration is suddenly changed to $-g$, so that the spaceship now decelerates. We now start part *ii* of the journey, that has constant deceleration $-g$, lasting until the spaceship reaches Proxima Centauri with speed exactly $v = 0$.

Having reached Proxima Centauri, the spaceship immediately turns around and returns to the Earth in the same way, so that it has acceleration $g = 9.8\,\mathrm{m\,s^{-2}}$ towards the Earth in part *iii* of the journey and $-g$ in part *iv*, ending with the spaceship reaching the Earth with speed $v = 0$.

Four-velocity is defined by

$$U_\mu = \frac{dx_\mu}{d\tau} = \gamma(u)(c, \mathbf{u}),$$

and four-acceleration by

$$A_\mu = \frac{dU_\mu}{d\tau} = \gamma^4(u)\left(\vec{\beta}\cdot\mathbf{a}, \mathbf{a} + \vec{\beta}\times(\vec{\beta}\times\mathbf{a})\right),$$

where τ is proper time, $\mathbf{u} = d\mathbf{x}/dt$, $\mathbf{a} = d\mathbf{u}/dt$, $\vec{\beta} = \mathbf{u}/c$ and $\gamma(u) = (1-\beta^2)^{-1/2}$. One can prove that

$$A^\mu A_\mu = \gamma^6(u)\left[a^2 - (\vec{\beta}\times\mathbf{a})\cdot(\vec{\beta}\times\mathbf{a})\right]$$

and

$$\frac{d\gamma}{dt} = \frac{\gamma^3\mathbf{u}\cdot\mathbf{a}}{c^2}.$$

a) Show that during part *i* of the journey, $\gamma^3(v)dv/dt = g$ and $d(\gamma v)/dt = \gamma^3 dv/dt$. Use these relations to show that

$$v = \frac{gt}{\sqrt{1 + \frac{g^2t^2}{c^2}}}$$

and

$$x = \frac{c^2}{g}\sqrt{1 + \frac{g^2t^2}{c^2}} - \frac{c^2}{g}.$$

b) How long does it take, measured in the inertial reference system of the Earth, from the spaceship leaves the Earth until it is back on Earth again? $1\text{pc} = 3.0856 \times 10^{16}\text{m}$ and $c = 2.9979 \times 10^8 \text{m s}^{-1}$.

c) Show that during part i of the journey, the relation between the time measured in the reference system of the spaceship (the spaceship's proper time) τ and the time measured in the Earth's reference system t is given by

$$\tau = \frac{c}{g}\text{arsinh}\left(\frac{gt}{c}\right),$$

and find how long time the whole journey to Proxima Centauri and back takes for an astronaut traveling in the spaceship.

Exercise 2

a) Observations show that the rotational velocity of spiral galaxies is constant at large distances. Show how this can be explained by a spherical dark halo with density $\propto r^{-2}$.

How does the density of stars in the disk decrease with distance from the centre of the galaxy?

b) Describe briefly the reasons why the inner planets in the solar system have higher density than the planets far from the Sun.

c) What is meant by luminosity distance? Mention some reasons why the luminosity distance to objects at cosmological distances is not the same distance as we would have measured using measuring rods (the proper distance)?

d) Mars uses 687 days to orbit the Sun once (sidereal period). How long does it take from Mars is nearest the Earth (in opposition) once until the next time it is nearest the Earth?

e) What is meant by the Schwarzschild radius?